Title and Abstract of Talks

The First workshop on Finite Element Methods for PDEs
April 6-7, 2016 (Farvardin 18-19, 1395)

University of Kurdistan

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The Fokker-Planck Operator as an Asymptotic Limit and its Finite Element Approximation

Mohammad Asadzadeh

Abstract

In these lectures we shall derive the Fokker-Plsanck operator from the particle transport equation by asymptotis expansion. Then we shall prove a priori error estimates for the standard Galerkin and streamline diffusion methods for the Fermi Pencil-beam equation obtained from a fully three dimensional Fokker-Planck equation in space $\mathbf{x} = (x, y, z)$ and velocity $\tilde{\mathbf{v}} = (\mu, \eta, \xi)$ variables. The Fokker-Flanck operator appears as a Laplace-Beltrami operator in the unit sphere. The Fermi operator, is obtained as a projection of the FP operator onto the tangent plane to the unit sphere at the pole $(1, 0, 0)$ and in the direction of $\mathbf{v}_0 = (1, \eta, \xi)$. Hence the Fermi equation, stated in three dimensional spatial domain $\mathbf{x} = (x, y, z)$, depends only on two velocity variables $\mathbf{v} = (\eta, \xi)$. This is "raison de etre" for the alternative phrasing: "two and one-half" dimensions that appears in some applications. We also sketch an a posteriori error estimate procedure which is employed in an adaptive algorithm for local mesh refinements in order to control error in the computed solution. Different numerical examples, in two space dimensions are justifying the theoretical results. Implementations show significant reduction of the computational error by using our adaptive algorithm and illustrate usefulness of the a posteriori error control.

On hp Finite Element for Vlasov-Poisson-Fokker-Planck System

Mohammad Asadzadeh

Abstract

We study the existence of unique solutions for nonlinear PDEs in the realm of the Vlasov-Poisson-Fokker-Planck systems. We introduce a unifying kinetic model preserving mass, moment and energy. Then we construct and analyze a numerical scheme for the multi-dimensional Vlasov-Poisson-Fokker-Planck system based on a backward-Euler (BE) approximation in time combined with a mixed finite element method for a discretization of the Poisson equation in the spatial domain and a discontinuous Galerkin (DG) finite element approximation in the phase-space variables for the Vlasov equation. The Fokker-Planck term is treated as a diffusion transport equation using the characteristic Galerkin approach. We prove the stability estimates and derive the optimal convergence rates depending upon the compatibility of the finite element meshes, used for the discretizations of the spatial variable in Poisson (mixed) and Vlasov (DG) equations, respectively. The error estimates for the Poisson equation are based on using Brezzi-Douglas-Marini (BDM) elements in L_2 and H^{-s}, $s > 0$, norms.

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Finite Element Approximation of Deterministic and Stochastic Evolution Problems.

Stig Larsson

Abstract

The goal of these lectures is to present an introduction to the theory of finite element approximation of stochastic evolution problems perturbed by noise. The equations are given a rigorous formulation based on the theory of semigroups of bounded linear operators and the theory of Ito integration in Hilbert space. The following topics will be covered:

1. Semigroup framework for the heat and wave equations.
2. Finite element approximation of the heat and wave equations.
3. Stochastic evolution problems, semigroup framework, regularity of solutions.
4. Finite element approximation of stochastic evolution problems, error estimates.

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Adaptive Methods for the Approximation of Solutions of Partial Differential Equations.

Omar Lakkis

Abstract

Adaptive methods for the approximation of solutions of partial differential equations (PDEs) often gain in efficiency and reliability when they are based on error control principles. Aposteriori error analysis provides a rigorous foundation for error control. The four lectures are dedicated to an overview of these methods.

1. In lecture one I will outline the state of the art for the most mature sector of linear elliptic PDEs. I review different approaches (though not all) and explain the ideas behind periods of convergence of adaptive methods.

2. In lecture two I will talk about parabolic PDEs with special focus on the elliptic reconstruction technique.

3. In lecture three I will take second order hyperbolic PDEs emphasising the differences with the parabolic case, as this is not a trivial extension thereof.

4. In lecture four I will look at nonlinear equations, including fully nonlinear ones such as the Monge-Ampre where adaptivity allows the approximation of viscosity solutions relatively efficiently.